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High overtones of Schwarzschild-de-Sitter quasinormal spectrum

Roman A. Konoplya and Alexander Zhidenko

Department of Physics, Dniepropetrovsk National University
St. Naukova 13, Dniepropetrovsk 49050, Ukraine
E-mail: konoplya_roma@yahoo.com, Zhidenko@ff.dsu.dp.ua

Abstract: We find the high overtones of gravitational and electromagnetic quasinormal spectrum of the Schwarzschild-de Sitter black hole. The calculations show that the real parts of the electromagnetic modes asymptotically approach zero. The gravitational modes show more peculiar behavior at large n: the real part oscillates as a function of imaginary even for very high overtones and these oscillations settles to some "profile" which just repeats itself with further increasing of the overtone number n. This lets us judge that $\mathrm{Re}\omega$ is not a constant as $n \to \infty$ but rather some oscillating function. The spacing for imaginary part $\mathrm{Im}\omega_{n+1} - \mathrm{Im}\omega_n$ for electromagnetic perturbations at high n slowly approach k_e as $n \to \infty$, where k_e is the surface gravity. In addition we find the lower QN modes for which the values obtained with numerical methods are in a very good agreement with those obtained through the 6th order WKB technique.

Keywords: AdS-CFT and dS-CFT Correspondence, Black Holes.

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Contents

1. Introduction

Quasinormal (QN) modes of black holes are of considerable interest recently because of their interpretation in ADS/CFT correspondence [1]–[7], and of possibility to detect gravitational waves from black holes [8]. Recently it has been observed that the quasinormal modes can play a fundamental role in Loop Quantum Gravity (LQG) [9]: for asymptotically flat black holes it has been found that the asymptotic value of the real part of the quasinormal frequency (i.e. when the overtone approaches infinity) coincides with the Barbero-Immirizi parameter, which must be fixed to predict the Bekenstein-Hawking formula for entropy within the framework of LQG. All this stimulated development of different approaches to calculation of quasinormal modes [10]. In particular, the analytical expression for asymptotically high QN modes of D-dimensional Schwarzschild black hole was obtained in [11, 12].

The asymptotic overtone quasinormal behavior was studied by Nollert [13] for the Schwarzschild black hole (see [5]), and in [6] for the Schwarzschild anti-de Sitter black hole. Yet there is no such study for Schwarzschild-de Sitter black hole (SdS BH), except for near extremal case [14]. By using the 6th order WKB technique the low lying modes of SdS BH were estimated in [15]. In the work [16], the low lying QN modes for gravitational perturbations of Schwarzschild-de Sitter black hole were obtained with the help of the Leaver method [17]. In [18] it was found the low lying QN modes for higher (> 4) dimensional Schwarzschild black hole with different values of lambda term; this includes cases of Schwarzschild, Schwarzschild-de Sitter, and Schwarzschild-anti-de Sitter black holes.

For the higher QN modes of the near extremal SdS BH, in [14] it was shown that in contrast to asymptotically flat SBH, the QN spectrum of SdS BH have oscillatory behavior: the real part oscillates as a function of imaginary. Recently there appeared a lot of works using different numerical and analytical approaches devoted to near extremal Schwarzschild-de-Sitter quasinormal spectrum [20].

In this paper we make numerical investigation of the high overtones of Schwarzschild-de-Sitter quasinormal spectrum. We show that at very high overtones the real parts of the gravitational QN modes oscillates as a function of imaginary part. Thus the real part of ω does not go to any constant limit and, thereby, the interpretation of the asymptotic QN frequency as those connected with the Barbero-Immirizi parameter in LQG is impossible. We also show that the real part of the electromagnetic QN modes approaches zero as an overtone number goes to infinity. At high overtones the QN spectrum shows rather peculiar behavior of some periodic weaving. The numerical results at high overtones are in a very good agreement with recent analytical (algebraic) equations which govern the asymptotic behavior, while the lower overtones are in a good agreement with those obtained earlier with the 6-th order WKB method.

The paper is organized as follows. In section 2 there are basic formulas of SdS background for Nollert technique. Section 3 is devoted to the low (first ten) overtones. Section 4 deals with very high overtones which let us judge about overtone asymptotic behavior.

2. Basic equations

The Schwarzschild-de-Sitter black hole is described by the metric

$$\frac{2M}{r} - \Lambda\frac{r^2}{3}\,, \qquad d\sigma^2 = d\theta^2 + \sin^2\theta d\phi^2\,, \tag{2.1}$$

where M is the black hole mass, Λ is the cosmological constant.

It is well known that the perturbation equations can be reduced to the Schrödinger wave-like equation

$$\left(\frac{d^2}{dr^{*2}} + \omega^2 - V(r^*)\right)\Psi(r^*) = 0\,, \tag{2.2}$$

by using the so-called tortoise coordinate:

$$dr^* = \frac{dr}{f(r)}\,. \tag{2.3}$$

Under the choice of the positive sign of the real part of ω, QNMs satisfy the following boundary conditions

$$\Psi(r^*) \sim C_\pm \exp(\pm i\omega r^*)\,, \qquad r \longrightarrow \pm\infty\,, \tag{2.4}$$

corresponding to purely in-going waves at the event horizon and purely out-going waves at the cosmological horizon.

The effective potential is given by

$$V(r) = f(r)\left(\frac{l(l+1)}{r^2} - \frac{2M\epsilon}{r^3}\right)\,, \tag{2.5}$$

where $\epsilon = 3$ for gravitational perturbations and $\epsilon = 0$ for electromagnetic ones.

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The appropriate Frobenius series are

$$\Psi(r^*) = \left(\frac{1}{r} - \frac{1}{r_e}\right)^{\rho_e} \left(\frac{1}{r_c} - \frac{1}{r}\right)^{-\rho_c} \left(\frac{1}{r} + \frac{1}{r_e + r_c}\right)^{\rho_c+\rho_e} \sum_{n\geq 0} a_n \left(\frac{1/r - 1/r_e}{1/r_c - 1/r_e}\right)^n , \quad (2.6)$$

where r_e is the event horizon, r_c is the cosmological horizon, ρ_e and ρ_c are determined by

$$e^{i\omega r^*} = \left(\frac{1}{r} - \frac{1}{r_e}\right)^{-\rho_e} \left(\frac{1}{r_c} - \frac{1}{r}\right)^{-\rho_c} \left(\frac{1}{r} + \frac{1}{r_e + r_c}\right)^{\rho_c+\rho_e} , \quad (2.7)$$

and one can find

$$\rho_e = \frac{i\omega}{2M\,(1/r_c - 1/r_e)\,(1/(r_c + r_e) + 1/r_e)}; \quad \rho_c = \frac{-i\omega}{2M\,(1/r_c - 1/r_e)\,(1/(r_c + r_e) + 1/r_c)}.$$

Substituting (2.6) into (2.2), we obtain the three-terms recurrent relation for a_n

$$a_{n+1}\alpha_n + a_n\beta_n + a_{n-1}\gamma_n = 0\,, \qquad n \geq 0\,, \quad \gamma_0 = 0\,, \quad (2.8)$$

where the coefficients α, β, γ have the form:

$$\begin{aligned}
\alpha_n &= \frac{r_c(r_c + 2r_e)(1 + n + 2\rho_e)^2}{r_c^2 + r_c r_e + r_e^2} + \frac{2r_c r_e(1 + n + 2\rho_e)}{r_c - r_e} i\omega \\
\beta_n &= -\frac{(n + 2\rho_e)(n + 2\rho_e + 1)(2r_c^2 + 2r_c r_e - r_e^2)}{r_c^2 + r_c r_e + r_e^2} - l(l+1) + \frac{r_c(r_c + r_e)}{r_c^2 + r_c r_e + r_e^2}\epsilon \\
\gamma_n &= \frac{r_c^2 - r_e^2}{r_c^2 + r_c r_e + r_e^2}((n + 2\rho_e)^2 - 1 - \epsilon)\,. \qquad (2.9)
\end{aligned}$$

Following Leaver [17] we are searching QNMs as the most stable roots of

$$\beta_n - \frac{\alpha_{n-1}\gamma_n}{\beta_{n-1} - \frac{\alpha_{n-2}\gamma_{n-1}}{\beta_{n-2} - \alpha_{n-3}\gamma_{n-2}/\ldots}} = \frac{\alpha_n\gamma_{n+1}}{\beta_{n+1} - \frac{\alpha_{n+1}\gamma_{n+2}}{\beta_{n+2} - \alpha_{n+2}\gamma_{n+3}/\ldots}}\,. \quad (2.10)$$

The infinite continued fraction on the right side of the equation (2.10) converges worse if the imaginary part of ω increases with respect to the real part. This problem was circumvented by Nollert [13]. He considered

$$R_N = \frac{\gamma_N}{\beta_N - \frac{\alpha_N\gamma_{N+1}}{\beta_{N+1} - \alpha_{N+1}\gamma_{N+2}/\ldots}} = \frac{\gamma_N}{\beta_N - \alpha_N R_{N+1}}\,. \quad (2.11)$$

Making use of the recurrence relation (2.11) one can find for large N:

$$R_N = C_0 + C_1 N^{-1/2} + C_2 N^{-1} + C_3 N^{-3/2} \ldots , \quad (2.12)$$

where

$$\begin{aligned}
C_0 &= \frac{r_c^2 - r_e^2}{r_c(r_c + 2r_e)}\,, \\
C_1 &= \pm\sqrt{\frac{2r_c^2 r_e + 5r_c r_e^2 + 2r_e^3 - 2(r_c^3 + r_e^3)r_e i\omega - 4(r_c + r_e)r_e^2 r_c i\omega}{r_c^3 + 4r_c r_e(r_c + r_e)}}\,, \qquad \mathrm{Re}(C)_1 > 0\,,
\end{aligned}$$

etc.

The series (2.12) converge for $|\omega|/N < A < \infty$, so we can use this approximation for R_N inside the continued fraction for some $N \gg -\mathrm{Im}(\omega) \sim n$. In practice to find an appropriate N we increase it until the result of the continued fraction calculations does not change. If one is limited by the near extremal SdS black hole the imaginary part, being proportional ro surface gravity, is still small in this limit, and, one can use the Frobenius method without Nollert modification [14] which includes expansion in N. For non extremal values of Λ the situation is more complex and we have to deal with Nollert technique as described above.

3. Lower overtones

Here we present results of calculation for first ten overtones for different values of Λ and l (see Appendix in this paper). It turned out that the lower overtones obtained here with the help of Leaver method are in a very good agreement with those obtained through the 6th order WKB method [23]. Thus for example for $\Lambda = 0.02$ and $l = 2$ gravitational perturbations from the 6th order WKB approach we have for the fundamental overtone $(n = 0)$ $\omega = 0.3384 - 0.817i$ while from the continued fraction we obtain $\omega = 0.33839143 - 0.08175645i$. For electromagnetic perturbations with $l = 1$ $n = 0$ we have $\omega = 0.2259 - 0.0842i$ and $\omega = 0.22594346 - 0.08410383i$ from the 6th order WKB formula and from continued fractions respectively. Note that the pure imaginary algebraically special value for $l = 2$ gravitational perturbations which corresponds to the 8th mode "move" to the 9th mode for $\Lambda = 0.02$ and to the higher mode for greater Λ. When the Λ is growing both the real and the imaginary parts of ω are decreasing, i.e. modes damp more slowly and oscillates with greater real frequency. In the near extremal regime, i.e. when the Λ term is close to its extremal value $1/9$ $(M = 1)$, the effective potential approaches the Pöschl-Teller potential and only several first modes are well described by the formula:

$$\omega b = -\left(n + \frac{1}{2}\right) i + \sqrt{l(l+1) - \frac{1}{4}} \tag{3.1}$$

for scalar and electromagnetic perturbations, and by

$$\omega b = -\left(n + \frac{1}{2}\right) i + \sqrt{(l+2)(l-1) - \frac{1}{4}} \tag{3.2}$$

for gravitational (axial) perturbations. Here

$$b = \frac{54M^3}{(r_c - 3M)(r_c + 6M)}, \qquad r_c \to 3M\,.$$

Note that the gravitational perturbations can be divided into the two kinds which can be treated separately: axial (symmetric with respect to the change $\varphi \to -\varphi$) and polar. In [23] it was shown both numerically and analytically that there is the isospectrality between these two kinds of perturbations, i.e. they both induce the same QN spectrum. That is why we treat here only the axial type of gravitational perturbations.

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4. High overtones

Finding of very high overtones is a time consuming procedure since the "length" of the continued fractions must be large enough. When we give for example the 100000th overtone that does not mean that we found all the previous 99999 modes; that would require an enormous amount of time. Yet we choose those modes to calculate which would characterize the structure of the quasinormal spectrum at high overtones.

4.1 Gravitational perturbations

We state that gravitational quasinormal spectrum of SdS black hole at asymptotically high overtones shows oscillatory behavior: *the real part of ω oscillates as a function of imaginary part and thus does not approach any constant value.* We have checked this for very high overtones (see for example figure 1 where computations performed up to $n \sim 165000$). The same behavior for the real part of ω was observed in the near extremal regime of Λ [14].

The imaginary part of ω is roughly proportional to n at large n and thereby can be approximately described by the formula:

$$\mathrm{Im}(\omega) \approx -k_e \left(n + \frac{1}{2}\right) i, \qquad n, \to \infty \,, \tag{4.1}$$

where k_e is the surface gravity at the event horizon.

Yet, this formula is not exact even for asymptotically high overtones since the spacing between nearby overtones $\mathrm{Im}(\omega_{n+1}) - \mathrm{Im}(\omega_n)$ shows very peculiar dependence on n (see as

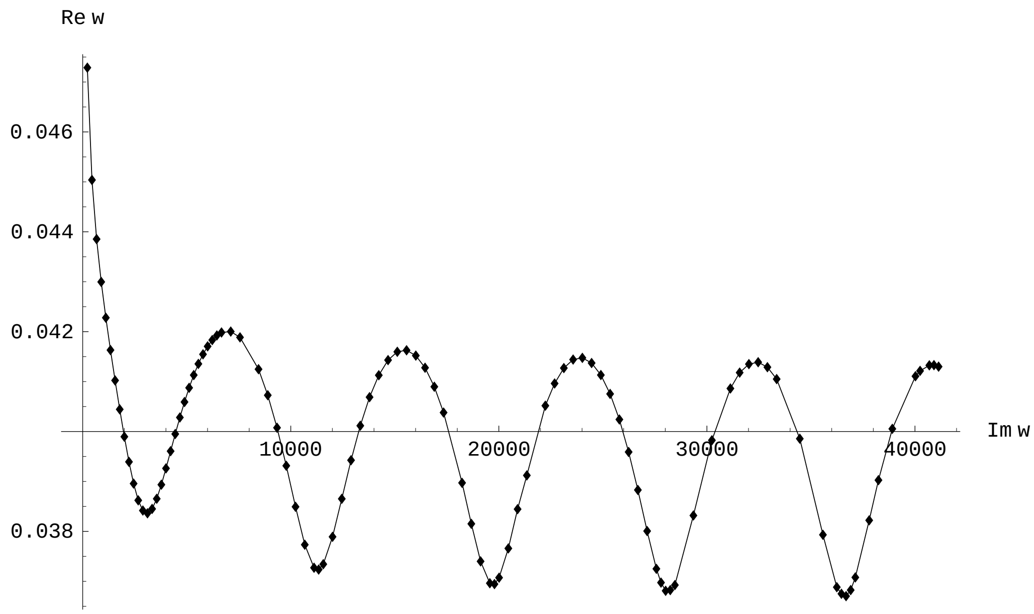

Figure 1: Real part of ω as a function of imaginary part for some values of $n = 10^3, 2 \cdot 10^3, \ldots$. In addition to this "large-scale" oscillation there is another sub-oscillations when considering nearby overtones (see for example figure 2). ($l = 2$ gravitational modes, $\Lambda = 0.02$).

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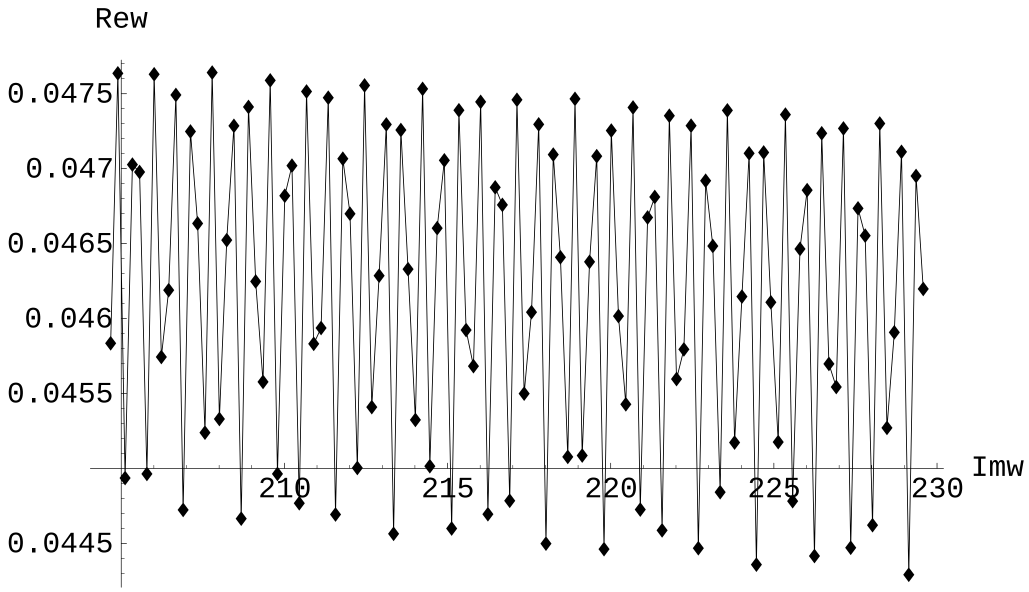

Figure 2: Real part of ω as a function of imaginary part ($l = 2$ gravitational modes, $\Lambda = 0.02$). From the first sight these modes can be misunderstood as just a numerical noise. Yet if pictured in a wider scale like on figure 4 they show *strict periodic structure*.

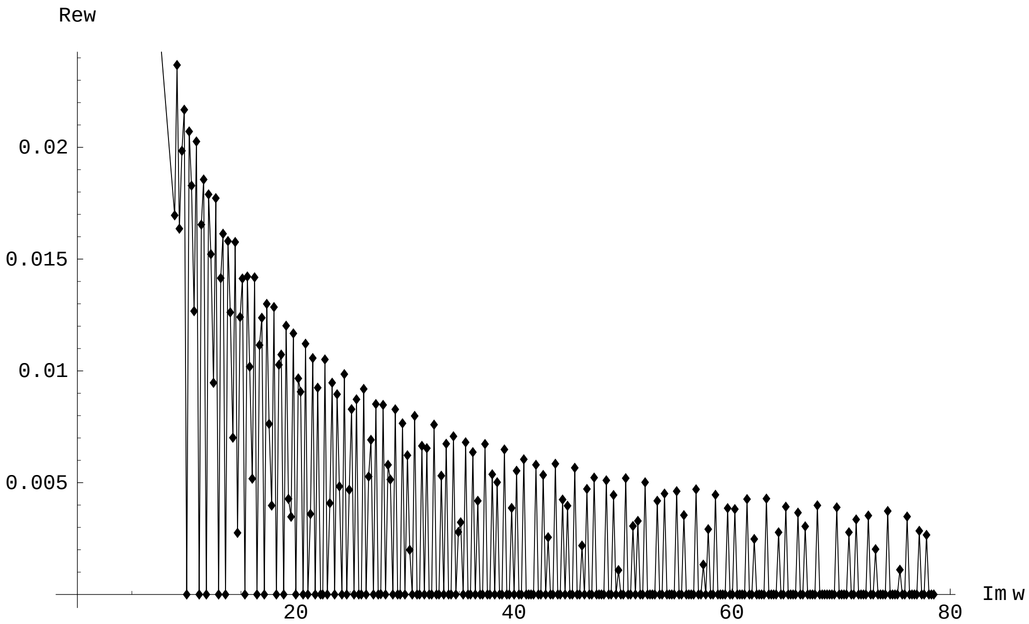

Figure 3: Real part of ω as a function of imaginary part ($l = 1$ electromagnetic modes, $\Lambda = 0.02$).

an example figure 4). From the first sight at figures 2 and 3 one could conclude that we have with numerical noise, yet if drawing the difference $\mathrm{Im}(\omega_{n+1}) - \mathrm{Im}(\omega_n)$ as a function of n in a greater scale (figure 4), we see that $\mathrm{Im}(\omega_{n+1}) - \mathrm{Im}(\omega_n)$ shows quite ordered repeated structures thus the behavior is *periodic*. It is important that these periodic weaves do not have tendency of damping, i.e. the spacing does not approach k_e however the average value of spacing over sufficiently large number of modes equals k_e:

$$\sum_{n=N_1}^{n=N_2} \frac{(\mathrm{Im}(\omega_{n+1}) - \mathrm{Im}(\omega_n))}{N_2 - N_1} \approx k_e\,, \qquad n \text{ is large}\,. \tag{4.2}$$

That is why the approximate formula (4.1) is valid. The true asymptotic formula, apparently, must have the form:

$$\mathrm{Im}(\omega) \approx -i\left(\left(n + \frac{1}{2}\right) k_e + f(n)\right), \qquad n \to \infty\,, \tag{4.3}$$

where $f(n)$ is some analytically unknown part consisting of small deviations from k_e similar to those shown on the figure 4. The average value of $f(n)$ in the sense of the formula (4.2) equals zero.

The oscillations of real part of ω as a function of imaginary part have very complicated form: the nearby overtones suffer from violate oscillations with lots of maximums and minimums similar to those shown on figure 2. These peaks form a larger wavy line like that shown on figure 1. Thus there is little hope to find a simple analytical expression which could describe these oscillations.

An important question is how we can judge whether the computed overtones are high enough to reflect the true asymptotic behavior?. We are sure that at a sufficiently large n the oscillations have a stable "profile" which just repeats itself with further increase of n. We can see the approaching of such a "final" profile on figure 1.

4.2 Electromagnetic perturbations

Electromagnetic perturbations show rather different behavior at high overtones: also there are oscillations of real part as a function of imaginary part, these oscillations damp when increasing the overtone number (see figure 4 as an example). Thus the real part of ω asymptotically approaches zero. We have checked already that at $n \sim 5000$ real part of ω is vanishing and we have not found QN modes with non-vanishing real part at higher n at all.

The imaginary part of electromagnetic QN modes shows the same behavior at high overtones as the gravitational modes do. That is, even though the imaginary part of ω is roughly proportional to n at large n, the spacing weaves as a function of n with average value $\sum_{n=N_1}^{n=N_2} \frac{(\mathrm{Im}(\omega_{n+1}) - \mathrm{Im}(\omega_n))}{N_2 - N_1} \approx k_e$, ($n$ is large) over sufficiently large quantity of modes. On contrary to gravitational perturbations, these weaves of the spacing of $\mathrm{Im}(\omega)$ damp and for very high n it is approaching the equidistant spectrum:

$$\mathrm{Im}(\omega) = -k_e\left(n + \frac{1}{2}\right) i, \quad n \to \infty\,. \tag{4.4}$$

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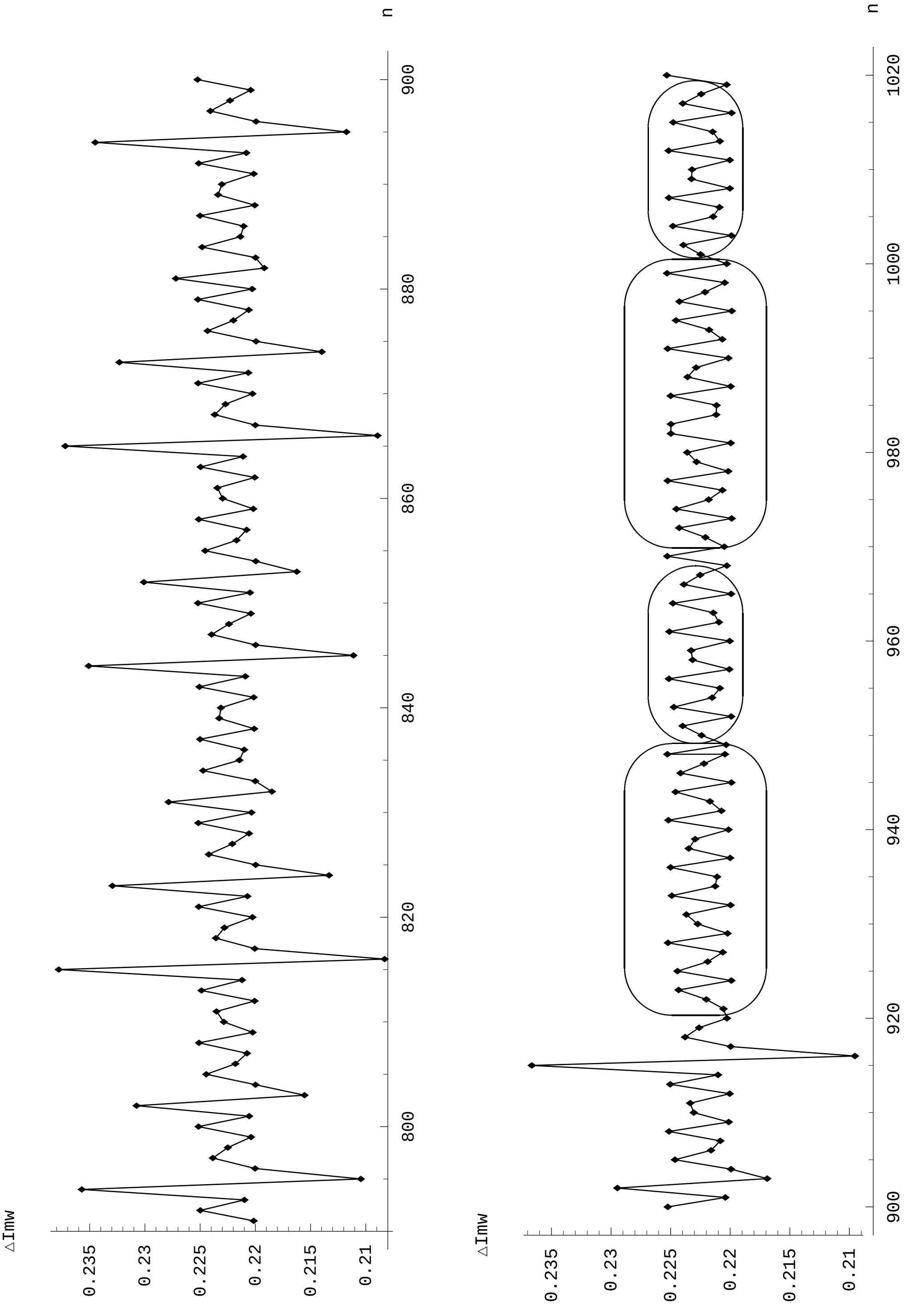

Figure 4: The spacing $\mathrm{Im}(\omega_{n+1}) - \mathrm{Im}(\omega_n)$ as a function of n for gravitational perturbations, ($\Lambda = 0.02$, $M = 1$, $l = 2$) for large n. The spacing shows peculiar behavior which is NOT a numerical noise, since it is strictly periodic (*continued*).

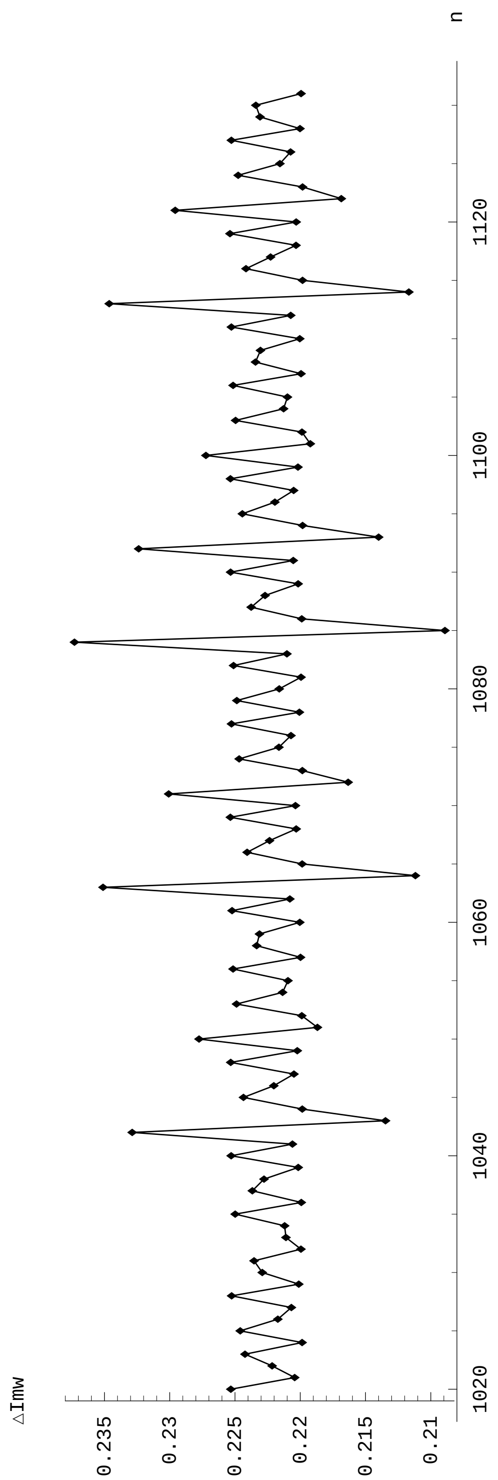

Figure 4: (*continued*) The spacing $\mathrm{Im}(\omega_{n+1}) - \mathrm{Im}(\omega_n)$ as a function of n for gravitational perturbations, ($\Lambda = 0.02$, $M = 1$, $l = 2$) for large n. The spacing shows peculiar behavior which is NOT a numerical noise, since it is strictly periodic.

5. Comparison with analytic formulas

After the first version of this work has been appeared [25], V. Cardoso, J. Natarrio and R. Schiappa [26] managed to find an analytical expression for $n \to \infty$ QN behavior. They found that the gravitational modes must obey the following algebraic equation

$$\cosh\left(\frac{\pi\omega}{k_e} - \frac{\pi\omega}{k_c}\right) + 3\cosh\left(\frac{\pi\omega}{k_e} + \frac{\pi\omega}{k_c}\right) = 0 \tag{5.1}$$

takes place for the asymptotically high ($n \to \infty$) gravitational modes (k_c is the surface gravity at the cosmological horizon.). While for electromagnetic modes they obtained the two possibilities

$$\omega = i\left(n + \frac{1}{2}\right)k_e \qquad \text{or} \qquad \omega = i\left(n + \frac{1}{2}\right)k_c\,. \tag{5.2}$$

First of all the numerical data shown on figure 1 are in a good agreement with the above asymptotic formula for gravitational perturbations and the more n the closer numerical values to its analytical values, and, thereby, the better the QN modes obey the algebraic equations (5.1). The difference is quite understood, since complete coincidence is only asymptotical. Yet the formula (5.1) does not predict the asymptotic behavior of imaginary part as a function of overtone number. It only connects the real and imaginary parts within one algebraic equation. From the approximate relation (4.2), one could expect that at some very high n one could observe the equdistant spectrum like $\omega = i(n+1/2)k_e$. We have not observed any signs of it, since the weaves of the spacing of imaginary part do not show any tendency to damping.

As to electromagnetic perturbations we observed that the asymptotic formula $\omega = i(n+1/2)k_e$ is really true. That is, the greater n the closer QN modes to the values $\omega = i(n+1/2)k_e$. This is difficult to see for not very high overtones, but the coincidence

Numerical result	Asymptotical formula
$0.039314 - 9783.906062i$	$0.037864 - 9783.907040i$
$0.038492 - 10228.624422i$	$0.037008 - 10228.625519i$
$0.037073 - 20012.425574i$	$0.036309 - 20012.426688i$
$0.037659 - 20457.144537i$	$0.037042 - 20457.145567i$
$0.038318 - 29351.507086i$	$0.037857 - 29351.507866i$
$0.039824 - 30240.943303i$	$0.039382 - 30240.943943i$
$0.039026 - 38245.869248i$	$0.038636 - 38245.869869i$
$0.040054 - 38912.946162i$	$0.039657 - 38912.946703i$
$0.041107 - 40024.740368i$	$0.040668 - 40024.740824i$
$0.041214 - 40247.099158i$	$0.040764 - 40247.099602i$
$0.041326 - 40691.816716i$	$0.040852 - 40691.817141i$
$0.041329 - 40914.175491i$	$0.040843 - 40914.175908i$
$0.041299 - 41136.534267i$	$0.040800 - 41136.534676i$

Table 1: Comparison of numerical results shown on figure 1 with the algebraic equation of (5.1).

Numerical result	Asymptotical formula
$\Lambda = 0.02$	
analytical	numerical
$0 - 2223.590486i(n = 10000)$	$0 - 2223.590586i$
$0 - 4447.180971i(n = 20000)$	$0 - 4447.181020i$
$0 - 44471.809715i(n = 200000)$	$0 - 44471.809720i$
$\Lambda = 0.05$	
$0 - 1761.123825i(n = 10000)$	$0 - 1761.123919i$
$0 - 3522.247649i(n = 20000)$	$0 - 3522.247699i$
$0 - 35222.476494i(n = 200000)$	$0 - 35222.476499i$

Table 2: Comparison of numerical results for electromagnetic $l = 1$ modes with the algebraic equation of 5.2.

of numerical and analytical results is very accurate for sufficiently high overtones. An instance of this approaching asymptotic regime is demonstrated on the table 1.

At the same time we have not found any QN modes close to $\omega = i(n + 1/2)k_c$ at high n. This agrees with alternative choise of *either* $\omega = i(n + 1/2)k_e$ *or* $\omega = i(n + 1/2)k_c$ predictions.

Thus our numerical data confirm the analytic results very well. This agreement between numerical and analytical results also stated in the paper [26].

6. Conclusion

We have shown that even at very large overtone number the real part of the gravitational quasinormal frequency of SdS black hole oscillates as a function of imaginary part and these oscillations do not have any tendency to damping, i.e. the real part of ω asymptotically does not approach any constant value. On contrary to gravitational modes, real part of electromagnetic modes oscillates as well but these oscillations are damping with the growing of the overtone number and the real part asymptotically approaches zero.

In the case of Schwarzschild black hole the interpretation of the asymptotic value for quasinormal frequency is known [9]: the real part of it coincides with the Barbero-Immirizi parameter. We see that such a direct correspondence should be impossible for Schwarzschild de Sitter background, since the asymptotic value for quasinormal frequency is not a constant. Thus the possible connection of the QN frequency with the Barbero-Immirizi parameter in LQG is still an open question for Schwarzschild de Sitter black hole.

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A. Lower overtones for electromagnetic and gravitational perturbations

QNMs (electromagnetic)

n	ωM ($\Lambda M^2 = 0.02$, $l = 1$)	ωM ($\Lambda M^2 = 0.02$, $l = 2$)
0	$0.22594346 - 0.08410383i$	$0.41502231 - 0.08614382i$
1	$0.19922380 - 0.26307549i$	$0.39900686 - 0.26200974i$
2	$0.16431511 - 0.46681587i$	$0.37040634 - 0.44852516i$
3	$0.13802472 - 0.68534232i$	$0.33705175 - 0.649553554i$
4	$0.11984831 - 0.90808056i$	$0.30647368 - 0.86242326i$
5	$0.10660509 - 1.13166067i$	$0.28135476 - 1.08205413i$
6	$0.09637285 - 1.35538999i$	$0.26123915 - 1.304842841i$
7	$0.08827708 - 1.57881240i$	$0.24497392 - 1.52897807i$
8	$0.08124026 - 1.80213441i$	$0.23156338 - 1.75362819i$
9	$0.07574049 - 2.02543170i$	$0.22028242 - 1.97841759i$
10	$0.07054630 - 2.24795338i$	$0.21062074 - 2.20317743i$
n	ωM ($\Lambda M^2 = 0.04$, $l = 1$)	ωM ($\Lambda M^2 = 0.04$, $l = 2$)
0	$0.20061096 - 0.07472608i$	$0.36722808 - 0.07623876i$
1	$0.18121373 - 0.23012834i$	$0.35602130 - 0.23064606i$
2	$0.15204473 - 0.40381212i$	$0.33469636 - 0.39145112i$
3	$0.12839123 - 0.59146416i$	$0.30755559 - 0.56281639i$
4	$0.11182397 - 0.78398818i$	$0.28092911 - 0.74463194i$
5	$0.09989826 - 0.97695141i$	$0.25829220 - 0.93318015i$
6	$0.09013754 - 1.17106475i$	$0.23991905 - 1.12512639i$
7	$0.08368248 - 1.36405979i$	$0.22500213 - 1.31861131i$
8	$0.07599793 - 1.55718153i$	$0.21269507 - 1.51274409i$
9	$0.07240721 - 1.75138650i$	$0.20235705 - 1.70710444i$
10	$0.06703704 - 1.94219038i$	$0.19350620 - 1.90149978i$
n	ωM ($\Lambda M^2 = 0.06$, $l = 1$)	ωM ($\Lambda M^2 = 0.06$, $l = 2$)
0	$0.17089050 - 0.06380912i$	$0.31181526 - 0.06477818i$
1	$0.15891839 - 0.19374672i$	$0.30497404 - 0.19515060i$
2	$0.13694710 - 0.33439045i$	$0.29124711 - 0.32841879i$
3	$0.11659604 - 0.48768053i$	$0.27176423 - 0.46763105i$
4	$0.10180201 - 0.64613954i$	$0.25033424 - 0.61479838i$
5	$0.09084565 - 0.80599862i$	$0.23083347 - 0.76843456i$
6	$0.08218828 - 0.96605417i$	$0.21454829 - 0.92579233i$
7	$0.07497191 - 1.12581591i$	$0.20119758 - 1.08497949i$
8	$0.06886092 - 1.28497681i$	$0.19016038 - 1.24500614i$
9	$0.06398420 - 1.44344850i$	$0.18088812 - 1.40538843i$
10	$0.06059324 - 1.60167622i$	$0.17296716 - 1.56589008i$

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n	ωM $(\Lambda M^2 = 0.09,\ l = 1)$	ωM $(\Lambda M^2 = 0.09,\ l = 2)$
0	$0.11053646 - 0.04153633i$	$0.20085020 - 0.04180306i$
1	$0.10759439 - 0.12478386i$	$0.19907499 - 0.12548885i$
2	$0.10125456 - 0.20875361i$	$0.19544199 - 0.20944161i$
3	$0.09045512 - 0.29524991i$	$0.18978198 - 0.29395362i$
4	$0.07890420 - 0.39018591i$	$0.18187205 - 0.37958230i$
5	$0.07225731 - 0.48374011i$	$0.17175342 - 0.46745082i$
6	$0.06380747 - 0.58344049i$	$0.16090315 - 0.55881594i$
7	$0.06207673 - 0.67726878i$	$0.15106524 - 0.65264576i$
8	$0.05339164 - 0.77726919i$	$0.14249130 - 0.74847390i$
9	$0.05559083 - 0.87187144i$	$0.13546731 - 0.84457777i$
10	$0.04543452 - 0.96887722i$	$0.12914704 - 0.94185889i$

n	ωM $(\Lambda M^2 = 0.11,\ l = 1)$	ωM $(\Lambda M^2 = 0.11,\ l = 2)$
0	$0.02545378 - 0.00961749i$	$0.04614431 - 0.00962073i$
1	$0.02542082 - 0.02885255i$	$0.04612347 - 0.02886223i$
2	$0.02535465 - 0.04808787i$	$0.04608172 - 0.04810386i$
3	$0.02525473 - 0.06732363i$	$0.04601892 - 0.06734573i$
4	$0.02512026 - 0.08656006i$	$0.04593488 - 0.08658793i$
5	$0.02495008 - 0.10579737i$	$0.04582932 - 0.10583057i$
6	$0.02474268 - 0.12503586i$	$0.04570190 - 0.12507377i$
7	$0.02449607 - 0.14427589i$	$0.04555216 - 0.14431764i$
8	$0.02420768 - 0.16351788i$	$0.04537960 - 0.16356233i$
9	$0.02387424 - 0.18276244i$	$0.04518356 - 0.18280800i$
10	$0.02349147 - 0.20201036i$	$0.04496330 - 0.20205483i$

QNMs (gravitational)

n	ωM $(\Lambda M^2 = 0.02,\ l = 2)$	ωM $(\Lambda M^2 = 0.02,\ l = 3)$
0	$0.33839143 - 0.08175645i$	$0.54311488 - 0.08449572i$
1	$0.31875867 - 0.24919663i$	$0.53074425 - 0.25536311i$
2	$0.28273218 - 0.42948412i$	$0.50701532 - 0.43205884i$
3	$0.24054151 - 0.62819192i$	$0.47483507 - 0.61839452i$
4	$0.20194822 - 0.84100966i$	$0.43911708 - 0.81624020i$
5	$0.16891621 - 1.06095994i$	$0.40465171 - 1.02429658i$
6	$0.13944328 - 1.28417153i$	$0.37401137 - 1.23949553i$
7	$0.11044868 - 1.50935542i$	$0.34769654 - 1.45900264i$
8	$0.07677592 - 1.73728467i$	$0.32524173 - 1.68092084i$
9	$0.00000000 - 1.98403566i$	$0.30595356 - 1.90413890i$
10	$0.05371227 - 2.27038762i$	$0.28919474 - 2.12803551i$

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n	ωM ($\Lambda M^2 = 0.04$, $l = 2$)	ωM ($\Lambda M^2 = 0.04$, $l = 3$)
0	$0.29889472 - 0.07329668i$	$0.48005752 - 0.07514635i$
1	$0.28584094 - 0.22172415i$	$0.47165827 - 0.22639484i$
2	$0.25999193 - 0.37709218i$	$0.45501064 - 0.38077311i$
3	$0.22627597 - 0.54558261i$	$0.43107569 - 0.54098604i$
4	$0.19342155 - 0.72690541i$	$0.40254385 - 0.70946230i$
5	$0.16520278 - 0.91558017i$	$0.37329553 - 0.88660299i$
6	$0.14087547 - 1.10757733i$	$0.34628879 - 1.07059758i$
7	$0.11886442 - 1.30135403i$	$0.32264813 - 1.25905540i$
8	$0.09700714 - 1.49561599i$	$0.30231712 - 1.45013447i$
9	$0.07243555 - 1.69317330i$	$0.28481977 - 1.64268182i$
10	$0.03639911 - 1.88991383i$	$0.26963306 - 1.83602924i$
n	ωM ($\Lambda M^2 = 0.06$, $l = 2$)	ωM ($\Lambda M^2 = 0.06$, $l = 3$)
0	$0.25328922 - 0.06304253i$	$0.40717516 - 0.06413956i$
1	$0.24574200 - 0.18979104i$	$0.40217056 - 0.19280739i$
2	$0.23007644 - 0.31915725i$	$0.39205277 - 0.32276933i$
3	$0.20669673 - 0.45518321i$	$0.37678925 - 0.45532936i$
4	$0.18068220 - 0.60121128i$	$0.35698877 - 0.59230260i$
5	$0.15723866 - 0.75477156i$	$0.33460484 - 0.73527072i$
6	$0.13729735 - 0.91211670i$	$0.31232419 - 0.88423367i$
7	$0.11998413 - 1.07114416i$	$0.29197277 - 1.03775443i$
8	$0.10425546 - 1.23089238i$	$0.27412339 - 1.19420006i$
9	$0.08916814 - 1.39091982i$	$0.25865044 - 1.35237729i$
10	$0.07372448 - 1.55104255i$	$0.24520634 - 1.51154481i$
n	ωM ($\Lambda M^2 = 0.09$, $l = 2$)	ωM ($\Lambda M^2 = 0.09$, $l = 3$)
0	$0.16261045 - 0.04136653i$	$0.26184253 - 0.04164389i$
1	$0.16078859 - 0.12415216i$	$0.26057158 - 0.12496881i$
2	$0.15704232 - 0.20711724i$	$0.25799755 - 0.20841194i$
3	$0.15114075 - 0.29047481i$	$0.25405408 - 0.29207761i$
4	$0.14267788 - 0.37469082i$	$0.24863818 - 0.37611940i$
5	$0.13118467 - 0.46102913i$	$0.24161615 - 0.46078805i$
6	$0.11824166 - 0.55218893i$	$0.23287254 - 0.54651859i$
7	$0.10767572 - 0.64571873i$	$0.22251601 - 0.63402624i$
8	$0.09695736 - 0.74179066i$	$0.21132493 - 0.72404510i$
9	$0.09027189 - 0.83757283i$	$0.20044112 - 0.81643184i$
10	$0.08075778 - 0.93460601i$	$0.19042690 - 0.91065927i$

n	ωM ($\Lambda M^2 = 0.11$, $l = 2$)	ωM ($\Lambda M^2 = 0.11$, $l = 3$)
0	$0.03726995 - 0.00961565i$	$0.06009145 - 0.00961888i$
1	$0.03724934 - 0.02884698i$	$0.06007662 - 0.02885667i$
2	$0.03720806 - 0.04807839i$	$0.06004694 - 0.04809452i$
3	$0.03714597 - 0.06730994i$	$0.06000235 - 0.06733248i$
4	$0.03706293 - 0.08654169i$	$0.05994279 - 0.08657059i$
5	$0.03695863 - 0.10577369i$	$0.05986815 - 0.10580890i$
6	$0.03683275 - 0.12500602i$	$0.05977830 - 0.12504745i$
7	$0.03668489 - 0.14423876i$	$0.05967310 - 0.14428631i$
8	$0.03651456 - 0.16347198i$	$0.05955236 - 0.16352551i$
9	$0.03632114 - 0.18270578i$	$0.05941586 - 0.18276513i$
10	$0.03610395 - 0.20194028i$	$0.05926336 - 0.20200521i$

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